| CHAPTER 3 |

Steel Beam Design

- **Learning Objectives**

- Understanding the concept of effective length, lateral and torsional restraints, destabilizing effects of loads.
- Calculation of the design loads for different limit states.
- Production of valid designs for simply supported steel beams: classification of sections, lateral torsional instability, moment capacity, buckling resistance moment, shear capacity, deflection check, web buckling resistance and web bearing resistance.

1. Steel Beam Design

1.1 Fully Restrained Beams (Clause 8.2)

Members in bending may be defined as either fully restrained or restrained at intervals. The design of a fully restrained beam and a beam restrained at intervals is quite different. The later one has to consider the ***Lateral Torsional Buckling (LTB)***.

Full lateral restraint is considered to be provided by the Code if the lateral restraining force between the floor and the compression flange can resist a lateral force of 2.5% of the maximum factored force in the compression flange of the member. For beams supporting either *in situ* or *precast concrete slabs*, it can generally be assumed that this criterion is satisfied.

1.2 Lateral Torsional Buckling of Beams (Clause 8.3)

The design of beams restrained at intervals (***LTB*** has to be taken into consideration) is treated in cl. 8.3 and will be discussed later.

1.3 Shear Capacity (Clause 8.2.1)

For loads parallel to the webs, the shear capacity V_c given below should not be less than the design shear force V.

$$V_c = \frac{p_y A_v}{\sqrt{3}} \geq V$$

The shear area (A_v) is defined in Clause 8.2.1 for different shaped sections as follows:

(a)	rolled I, H and channel sections	tD
(b)	welded I sections	td
(c)	rolled and welded rectangular hollow sections	2td
(d)	rolled and welded rectangular T sections	t (D – T)
(e)	circular hollow sections	0.6A
(f)	solid rectangular sections	0.9A
(g)	others	$0.9A_o$

where t is the web thickness;
T is the flange thickness;
B is the overall breadth;

D is the overall depth;
d is the depth of the web;
A is the area of the section;
A_o is the area of the rectilinear element of the section which has the largest dimension in the direction parallel to the load.

2. Beam in Bending (Compression Flange Fully Restrained)

2.1 Moment Capacity with Low Shear Load (Clause 8.2.2.1)

If the shear load (V) is less that 60% of the shear capacity (V_c), the effect of shear on the bending capacity is small and may be ignored and it is classified as low shear load.

For Class 1 plastic and Class 2 compact sections (i.e. where the full plastic moment can be developed), the moment capacity (M_c) is given by

$$M_c = p_y S \leq 1.2\, p_y Z \qquad \text{(Expression 1)}$$

The restriction applied to moment capacity, i.e. $M_c \leq 1.2\, p_y Z$, is to ensure that plasticity does not occur at working load.

For Class 3 semi-compact sections (i.e. elastic moment can be developed but local buckling may prevent development of the full plastic moment), the moment capacity (M_c) is given by

$$M_c = p_y Z \text{ or } M_c = p_y S_{eff} \qquad \text{(Expression 2)}$$

For Class 4 slender sections (i.e. local buckling may prevent development of the full elastic moment),

$$M_c = p_y Z_{eff} \text{ or } M_c = p_{yr} Z \qquad \text{(Expression 3)}$$

where S is the plastic modulus;
Z is the elastic modulus;
S_{eff} is the effective plastic modulus;
Z_{eff} is the effective elastic modulus;
P_{yr} is the design strength reduced for slender sections.

Class 1 and Class 2 sections with low shear load would be encountered in most practical cases.

2.2 Moment Capacity with High Shear Load (Clause 8.2.2.2)

If the shear load is greater than 60% of the shear capacity, the effect of shear should be taken into account as given in Clause 8.2.2.2.

For Class 1 plastic and Class 2 compact sections, the moment capacity (M_c) is given by

$$M_c = p_y\,(S - \rho S_v) \leq 1.2\,p_y\,(Z - \rho S_v/1.5) \qquad \text{(Expression 4)}$$

For Class 3 semi-compact sections, the moment capacity (M_c) is given by

$$M_c = p_y\,(Z - \rho S_v/1.5) \text{ or } M_c = p_y\,(S_{eff} - \rho S_v/1.5) \qquad \text{(Expression 5)}$$

For Class 4 slender sections

$$M_c = p_y\,(Z_{eff} - \rho S_v/1.5) \qquad \text{(Expression 6)}$$

where S_v is the plastic modulus of shear area A_v;

ρ is given by $\left(\dfrac{2V}{V_c} - 1\right)^2$.

3. Web Bearing and Web Buckling

3.1 Web Bearing (clause 8.4.10.5)

HK Code allows dispersion of 1:2.5 to the root radius line but there is no enhancement of p_y.

The bearing capacity P_{bw} of unstiffened web at the web-to-flange connection is given by:

$$P_{bw} = (b_1 + nk)\,t\,p_{yw}$$

in which
at the ends of a member:

$n = 2 + 0.6\,b_e/k \leq 5$

at other locations

$n = 5$

For rolled I or H sections, $k = T + r$

For welded I or H sections, $k = T$

where
b₁ is the stiff bearing length (cl. 8.4.10.2)
$\boldsymbol{b_e}$ is the distance to the nearer end of the member from the end of the stiff bearing.
t is the web thickness
T is the flange thickness
r is the root radius
$\boldsymbol{p_{yw}}$ = design strength of web.

The stiff bearing length $\boldsymbol{b_1}$ is defined as the length which cannot deform appreciably in bending. The dispersion of the load is taken as 45° through solid materials.

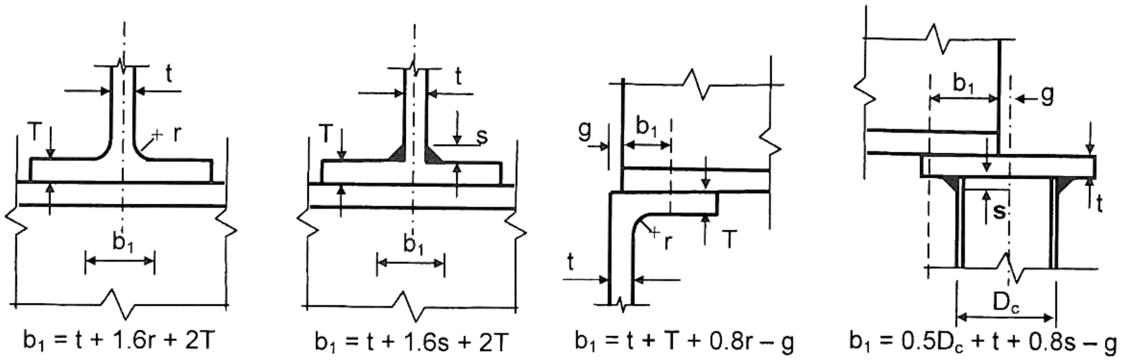


Figure 1 – Extract of Figure 8.3 of the HK Code[1]

3.2 Web Buckling of Unstiffened Web (clause 8.4.10.6)

When the flange, through which the load or reaction is applied, is effectively restrained against both:
(a) Rotation relative to the web;
(b) Lateral movement relative to other flange.

The buckling resistance $\boldsymbol{P_x}$ is:

$$P_x = \frac{25\varepsilon t}{\sqrt{(b_1 + nk)d}} P_{bw} \quad \text{for } a_e \geq 0.7d$$

and $$P_x = \frac{a_e + 0.7d}{1.4d} \cdot \frac{25\varepsilon t}{\sqrt{(b_1 + nk)d}} P_{bw} \quad \text{for } a_e < 0.7d$$

When the condition (a) or (b) is not met, the buckling resistance of the web is reduced to P_{xr}

$$P_{xr} = \frac{0.7d}{L_E} \cdot P_x$$

where a_e = distance from the load or reaction to the nearer end of the member.
L_E = effective length of the web acting as a compression member.

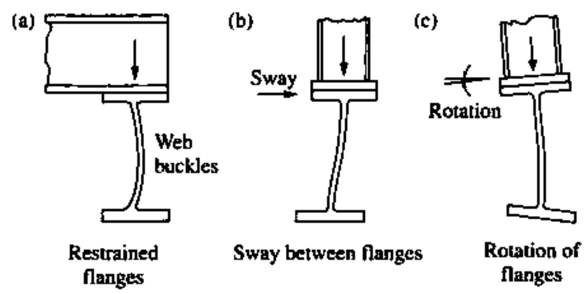


Figure 2 – Restrained Flanges, Sway Between Flanges and Rotation of Flanges (extracted from ref. 2)

4. Procedures for the Design of Restrained Beam

(a) Determine dead load and imposed load.
(b) Work out factored moment and shear force.
(c) Determine member size using $S_x = M_x / p_y$.
(d) Determine the section class.
(e) Check bending and shear capacities of beam.
(f) Check web buckling and web bearing if necessary.
(g) Check deflection.

5. Lateral Torsional Buckling of Beams (Compression Flange NOT Fully Restrained, Restrained at Intervals)

In addition to the checks required for fully restrained beams, members restrained at intervals should be checked for ***Lateral Torsional Buckling*** (***LTB***) between restraints (refer to clause 8.3). Figure 3 and Figure 4 show the deformations of beam with Lateral Torsional Buckling.

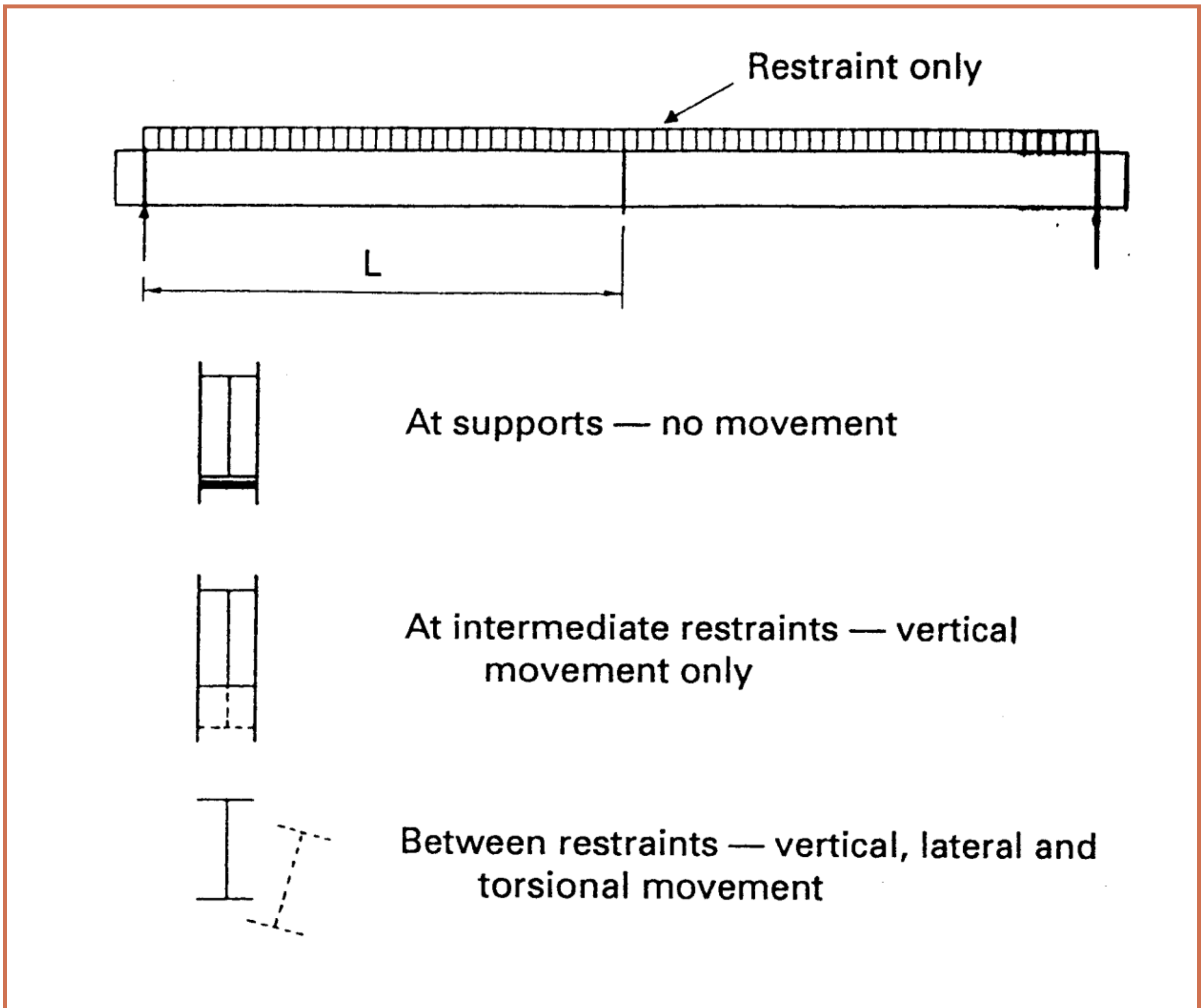


Figure 3 – Lateral Torsional Buckling

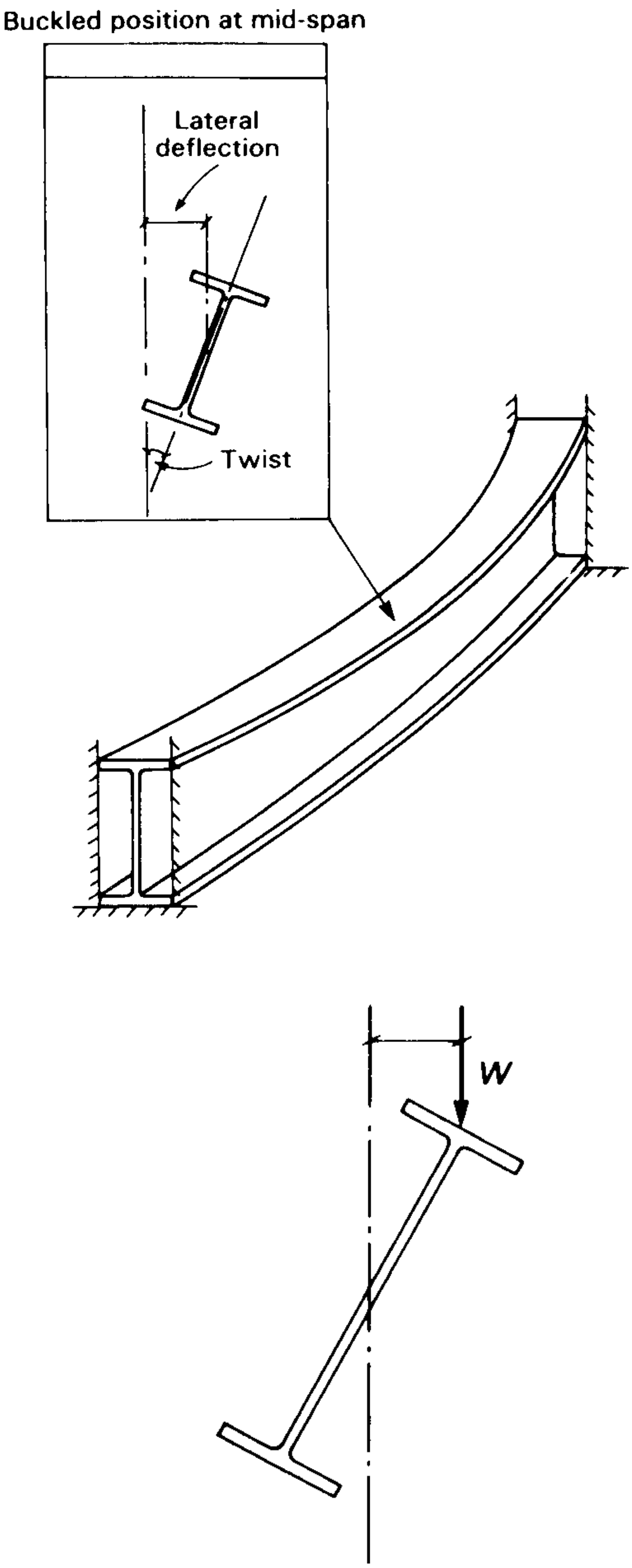


Figure 4 – Lateral Torsional Buckling (extracted from ref. 3)

5.1 Normal and Destablilizing Loads (clause 8.3.3)

A destabilizing loading condition should be assumed when considering effective length of beams with ***dominant loads applied to the top flange*** and ***with both the loads and the flange free to deflect and rotate relative to the shear center*** of the cross-section.

5.2 Effective Length for Lateral Torsional Buckling (clause 8.3.4)

5.2.1 Simple beams without intermediate lateral restraints.

(a) For a beam under normal loading conditions with its compression flange restrained against lateral movement at the end supports, but free to rotate on plan with ends under nominal torsional restraint about the longitudinal axis of the beam at end supports, the effective length is 1.0 of the span of the beam, i.e.

$$L_E = L_{LT}$$

(b) For a beams under normal loading conditions with the compression flange fully restrained against rotation on plan at its end supports,

$$L_E = 0.8L_{LT}$$

(c) For a beam under normal loading conditions with compression flanges unrestrained against lateral movement at end supports and with both flanges free to rotate on plan,

$$L_E = 1.2L_{LT} + 2D$$

(d) For beams under destabilizing loads, the effective length should be multiply by a factor of 1.2.

where L_{LT} is the segment length between lateral restraints under consideration.

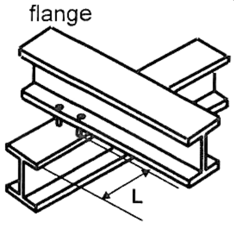
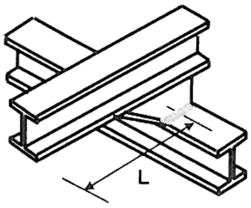
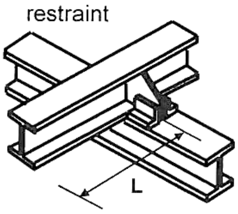
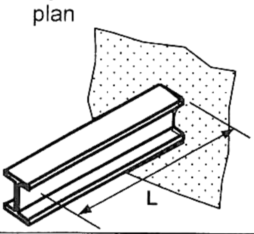
5.2.2 Beams with intermediate lateral restraints.

For simple beams with adequate intermediate lateral restraints, the effective length L_E is normally taken as $1.0L_{TL}$ for a normal load or $1.2L_{LT}$ for a destabilizing load.

5.2.3 Cantilever

For determining the effective lengths of cantilever beams, refer to Table 8.1 of the Code.

Table 8.1 - Effective length of cantilever without intermediate restraints

Restraint condition		Loading condition	
At support	At tip	Normal	Destabilizing
a) continuous, with lateral restraint to top flange  	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	3.0L 2.7L 2.4L 2.1L	7.5L 7.5L 4.5L 3.6L
b) continuous, with partial torsional restraint  	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	2.0L 1.8L 1.6L 1.4L	5.0L 5.0L 3.0L 2.4L
c) continuous, with lateral and torsional restraint  	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	1.0L 0.9L 0.8L 0.7L	2.5L 2.5L 1.5L 1.2L
d) restrained laterally and torsionally and against rotation on plan  	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	0.8L 0.7L 0.6L 0.5L	1.4L 1.4L 0.6L 0.5L

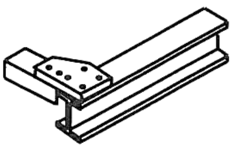
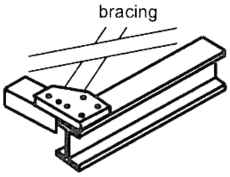
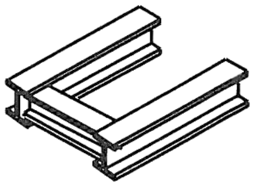
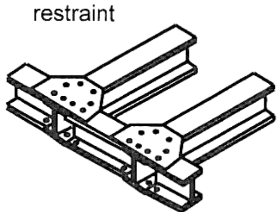
Tip restraint conditions			
1) Free  (not braced on plan)	2) Lateral restraint to top flange  (braced on plan in one or more bay)	3) Torsional restraint  (not braced on plan)	4) Lateral and torsional restraint  (braced on plan in one or more bay)

Table 1 – Extract of Table 8.1 of the HK Code[1]

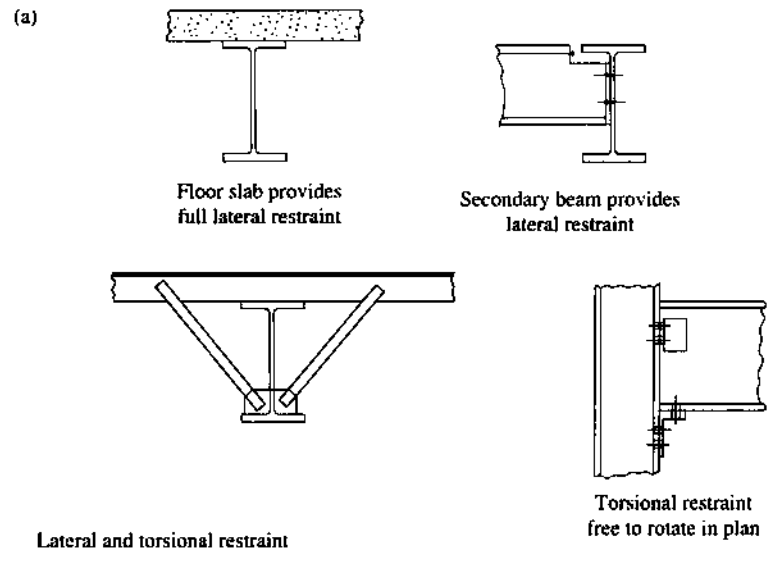


Figure 5 – Restraints to Beams (extracted from ref. 2)

5.3 Moment Resistance to Lateral Torsional Buckling (Buckling Resistance Moment M_b)

5.3.1 Code's Requirements

In each segment of a beam, the buckling resistance moment $\boldsymbol{M_b}$ should satisfy the following.

$$m_{LT} M_x \leq M_b \text{ and } M_x \leq M_{cx}$$

where $\boldsymbol{m_{LT}}$ is the *equivalent uniform moment factor* for lateral torsional buckling of simple beams obtained from ***Table 8.4*** of the Code. Conservatively it can be taken as unity (*1*). For cantilevers, $\boldsymbol{m_{LT}}$ is equal to 1.

$\boldsymbol{M_x}$ = Maximum bending moment along the beam.

For Class 1 plastic and Class 2 compact sections, $\boldsymbol{M_b = p_b S_x}$.
For Class 3 semi-compact sections, $\boldsymbol{M_b = p_b Z_x}$ or $\boldsymbol{M_b = p_b S_{eff}}$.

For Class 4 slender sections, $\boldsymbol{M_b = p_b Z_{eff}}$ or $M_b = p_b \cdot \dfrac{p_{yr}}{p_y} \cdot Z_x$

$\boldsymbol{p_b}$ is the buckling strength of the beam, determined from ***Table 8.3a*** for hot-rolled sections using a suitable *equivalent slenderness* λ_{LT} in clause 8.3.5.3 and relevant design strength $\boldsymbol{p_y}$.

5.3.2 Design Procedures for Unrestained Beam Segment / Beam

(a) Determine dead load and imposed load.

(b) Work out factored moment and shear force.

(c) Determine member size using $S_x = M_x / p_y$ as a guide. ***Heavier section is normally required as p_b is usually smaller than p_y.***

(d) Determine the section classification.

(e) Check buckling resistance moment of beam.

- Classify the design load as ***normal*** load or ***destabilizing*** load. Estimate the effective length L_E of the unrestrained compression flange using the rules from clause 8.3.4. Minor axis slenderness $\lambda = L_E/r_y$, where r_y = radius of gyration for the y-y axis.

- Calculate the equivalent slenderness λ_{LT}

  $$\lambda_{LT} = uv\lambda\sqrt{\beta_W}$$

  where
  u = buckling parameter allowing for torsional resistance. It can be obtained from Appendix 8.2 of the Code or conservatively equal to 0.9 for hot-rolled sections or taking from the section table.

  v = slenderness factor given by, $v = \dfrac{1}{\left(1+0.05(\lambda / x)^2\right)^{0.25}}$

  x = is the torsional index from Appendix 8.2 of the Code or conservatively equal to D/T or taking from the section table.

  $\beta_w = 1.0$ for Class 1 plastic section and Class 2 compact section.

- Read out the buckling strength p_b from table 8.3a of the Code.

- Calculate the buckling resistance moment $M_b = p_b S_x$.

- Determine the equivalent uniform moment factor m_{LT} from Table 8.4a or Table 8.4b of the Code.

- Calculate the design moment $m_{LT} M_x$ for checking ***LTB***.

- Compare $m_{LT} M_x$ with M_b. Check if $M_b > m_{LT} M_x \Rightarrow$ O.K.

(f) Check the shear capacity of beam.

(g) Check web buckling and web bearing if necessary.

(h) Check deflection.

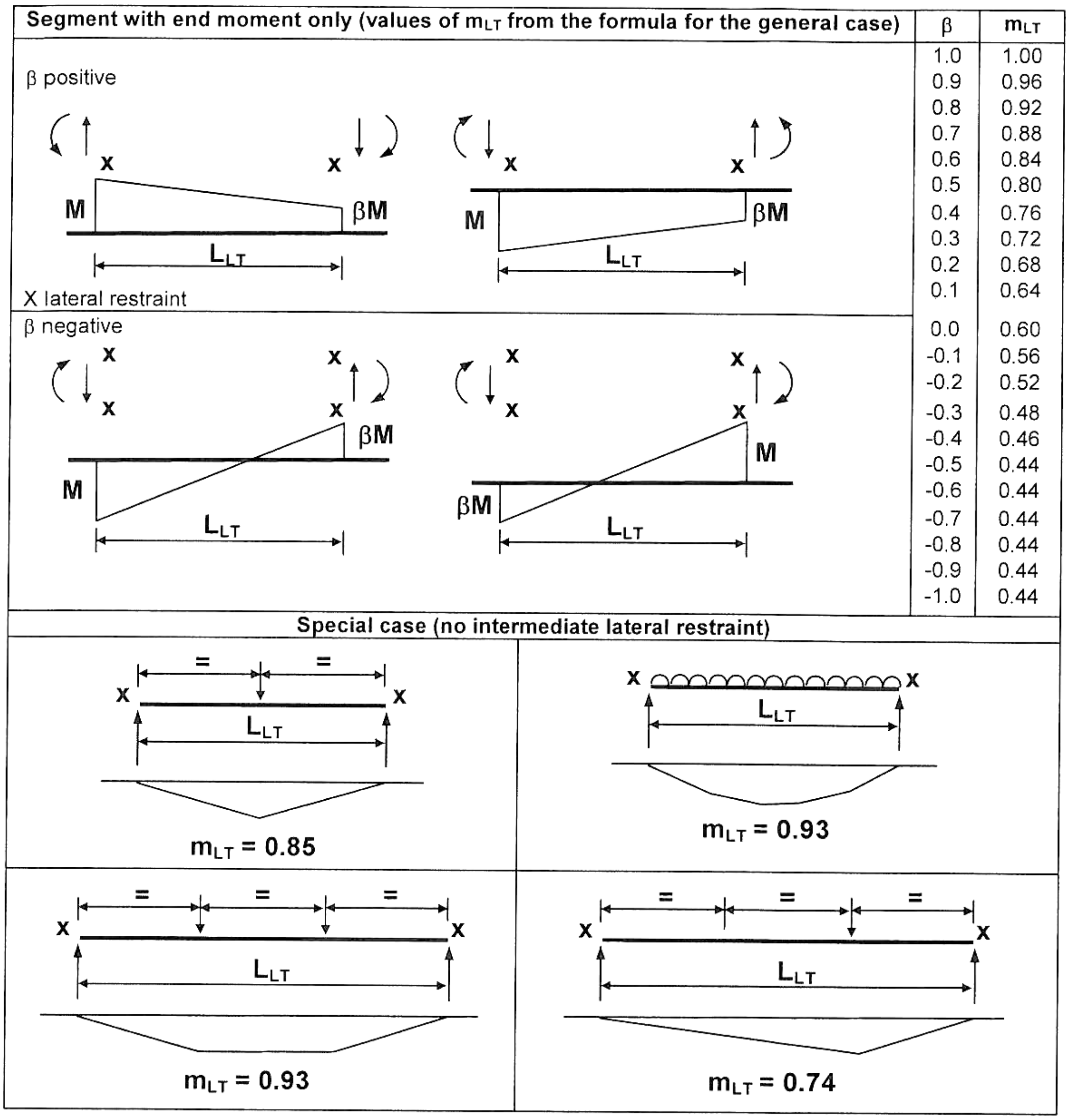
Table 8.3a - Bending strength p_b (N/mm^2) for rolled sections

λ_{LT}	Steel grade and design strength p_y (N/mm^2)														
	S275					S355					S460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
25	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
30	235	245	255	265	275	315	325	335	345	355	395	403	421	429	446
35	235	245	255	265	273	307	316	324	332	341	378	386	402	410	426
40	229	238	246	254	262	294	302	309	317	325	359	367	382	389	404
45	219	227	235	242	250	280	287	294	302	309	340	347	361	367	381
50	210	217	224	231	238	265	272	279	285	292	320	326	338	344	356
55	199	206	213	219	226	251	257	263	268	274	299	305	315	320	330
60	189	195	201	207	213	236	241	246	251	257	278	283	292	296	304
65	179	185	190	196	201	221	225	230	234	239	257	261	269	272	279
70	169	174	179	184	188	206	210	214	218	222	237	241	247	250	256
75	159	164	168	172	176	192	195	199	202	205	219	221	226	229	234
80	150	154	158	161	165	178	181	184	187	190	201	203	208	210	214
85	140	144	147	151	154	165	168	170	173	175	185	187	190	192	195
90	132	135	138	141	144	153	156	158	160	162	170	172	175	176	179
95	124	126	129	131	134	143	144	146	148	150	157	158	161	162	164
100	116	118	121	123	125	132	134	136	137	139	145	146	148	149	151
105	109	111	113	115	117	123	125	126	128	129	134	135	137	138	140
110	102	104	106	107	109	115	116	117	119	120	124	125	127	128	129
115	96	97	99	101	102	107	108	109	110	111	115	116	118	118	120
120	90	91	93	94	96	100	101	102	103	104	107	108	109	110	111
125	85	86	87	89	90	94	95	96	96	97	100	101	102	103	104
130	80	81	82	83	84	88	89	90	90	91	94	94	95	96	97
135	75	76	77	78	79	83	83	84	85	85	88	88	89	90	90
140	71	72	73	74	75	78	78	79	80	80	82	83	84	84	85
145	67	68	69	70	71	73	74	74	75	75	77	78	79	79	80
150	64	64	65	66	67	69	70	70	71	71	73	73	74	74	75
155	60	61	62	62	63	65	66	66	67	67	69	69	70	70	71
160	57	58	59	59	60	62	62	63	63	63	65	65	66	66	67
165	54	55	56	56	57	59	59	59	60	60	61	62	62	62	63
170	52	52	53	53	54	56	56	56	57	57	58	58	59	59	60
175	49	50	50	51	51	53	53	53	54	54	55	55	56	56	56
180	47	47	48	48	49	50	51	51	51	51	52	53	53	53	54
185	45	45	46	46	46	48	48	48	49	49	50	50	50	51	51
190	43	43	44	44	44	46	46	46	46	47	48	48	48	48	48
195	41	41	42	42	42	43	44	44	44	44	45	45	46	46	46
200	39	39	40	40	40	42	42	42	42	42	43	43	44	44	44
210	36	36	37	37	37	38	38	38	39	39	39	40	40	40	40
220	33	33	34	34	34	35	35	35	35	36	36	36	37	37	37
230	31	31	31	31	31	32	32	33	33	33	33	33	34	34	34
240	28	29	29	29	29	30	30	30	30	30	31	31	31	31	31
250	26	27	27	27	27	28	28	28	28	28	29	29	29	29	29
λ_{L0}	37.1	36.3	35.6	35.0	34.3	32.1	31.6	31.1	30.6	30.2	28.4	28.1	27.4	27.1	26.5

λ_{L0} is the maximum slenderness ratio of the member having negligible buckling effect.

Table 2 – Extract of Table 8.3a of the Code[1]

Table 8.4a - Equivalent uniform moment factor m_{LT} for lateral-torsional buckling of beams under end moments and typical loads



Segment with end moment only (values of m_{LT} from the formula for the general case)

β	m_{LT}
1.0	1.00
0.9	0.96
0.8	0.92
0.7	0.88
0.6	0.84
0.5	0.80
0.4	0.76
0.3	0.72
0.2	0.68
0.1	0.64
0.0	0.60
-0.1	0.56
-0.2	0.52
-0.3	0.48
-0.4	0.46
-0.5	0.44
-0.6	0.44
-0.7	0.44
-0.8	0.44
-0.9	0.44
-1.0	0.44

Table 3 – Extract of Table 8.4a of the HK Code[1]

Table 8.4b - Equivalent uniform moment factor m_{LT} for lateral-torsional buckling of beams under non-typical loads

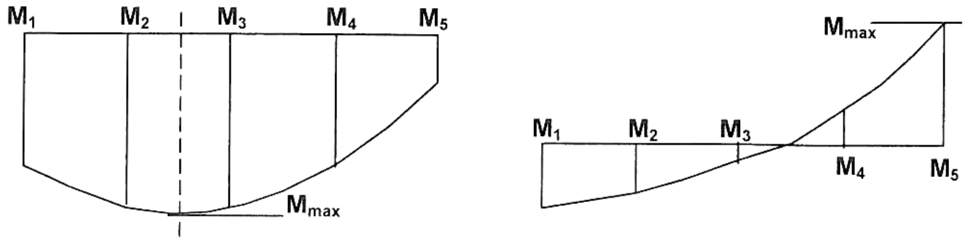
General case (segment between intermediate lateral restraint)

For beams: $m_{LT} = 0.2 + \frac{0.15M_2 + 0.5M_3 + 0.15M_4}{M_{max}}$ but $m_{LT} \geq 0.44$ All moment are taken as positive. The moment M_2 and M_4 are the values at the quarter points, M_3 is the value at mid-length and M_{max} is the maximum moment in the segment.
For cantilevers without intermediate lateral restraint, m_{LT} = 1.00

Table 4 – Extract of Table 8.4b of the HK Code[1]

Design Procedures of Structural Steel Beams in Bending with Low Shear Load (Plastic / Compact Sections)

Steel Beam Design for Bending with Low Shear Load

(Plastic/Compact sections)

Compression Flange Fully Restrained, e.g. with concrete slab on top

Moment Capacity:
$M_c = p_y{*}S_x$
$\leq 1.2p_y{*}Z_x$

If $M_c \geq$ Design Moment $\Rightarrow$ Bending O.K.

Finished

Compression Flange **NOT** Fully Restrained or Restrained at Intervals.

Set $\beta_w = 1$ and determine L_E

1. Read from the section table, the values of r_y, u, x, S_x
2. Find $\lambda = L_e / r_y$
3. Find λ/x and make use of the formula, $v = \dfrac{1}{\left(1 + 0.05(\lambda / x)^2\right)^{0.25}}$

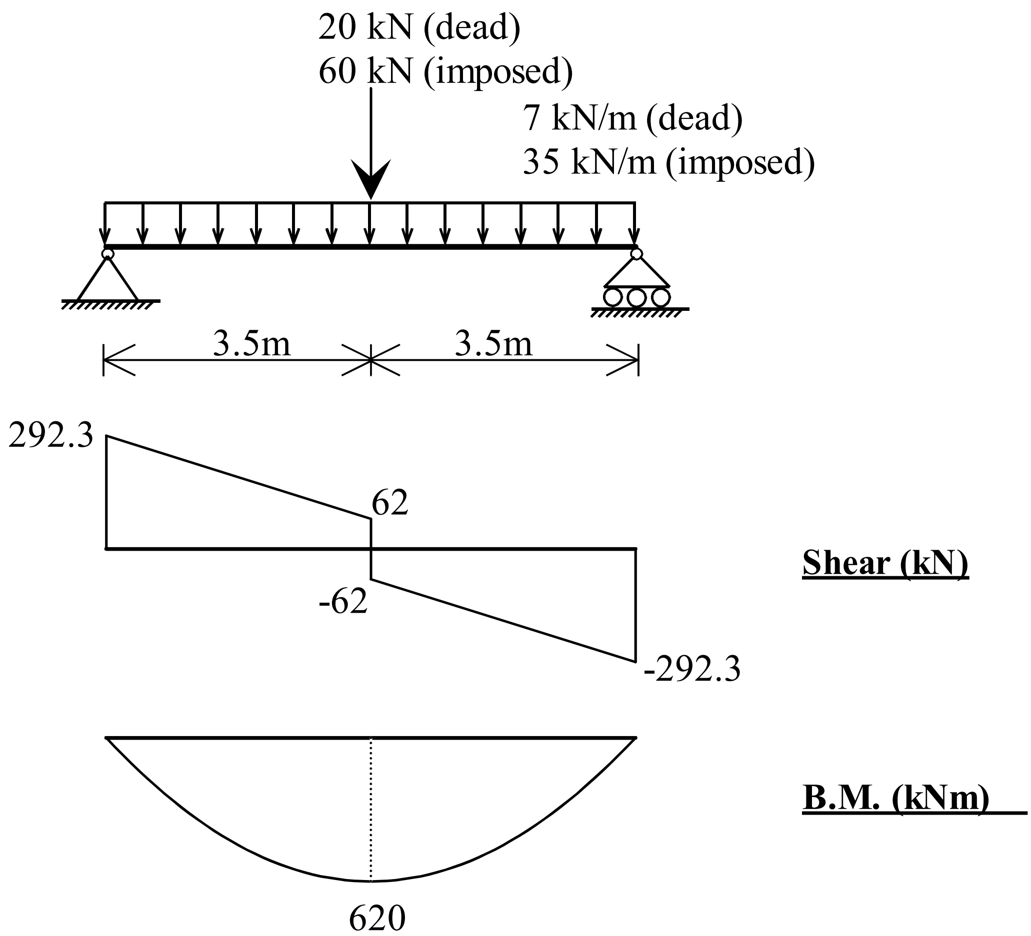
   to determine v.
4. Determine p_y based on the flange thickness.
5. Find $\lambda_{LT} = uv\lambda\sqrt{\beta_w}$, and make use of Table 8.3a to find p_b.
6. Calculate $M_b = p_b{*}S_x$
7. Determine m_{LT} from Table 8.4a or Table 8.4b.
8. Find the design moment $M = m_{LT}{*}M_x$

If $M_b > m_{LT}{*}M_x \Rightarrow$ Bending O.K.

Finished

Example 1 (Beam with Full Lateral Restraint)

A steel beam is carrying the characteristic loads as shown below and full lateral restraint is provided to the compression flange of the steel beam. Select an appropriate steel section of grade S355. Check the moment capacity, the shear capacity, deflection limit, web buckling resistance and web bearing resistance at the supports. Assume the width of stiff bearing at the supports is 160 mm and the flange is effectively restrained against both rotation relative to the web and lateral movement relative to other flange. Take $\boldsymbol{a_e}$ = 80 mm and $\boldsymbol{b_e}$ = 0 mm.



Solution

Factored Loads: P.L. (20*1.4) + (60*1.6) = 124 kN
UDL (7*1.4) + (35*1.6) = 65.8 kN/m

Applied Moment $M_x = \frac{124*7}{4} + \frac{65.8*7^2}{8} = 217 + 403 = 620\,\text{kNm}$

Assume the flange thickness T is less than 16 mm, p_y = 355 N/mm^2

The plastic section modulus required,

$$S_x = \frac{M_x}{p_y} = \frac{620x10^6}{355} = 1746000\text{mm}^3 = 1746\,\text{cm}^3$$

Try 533 x 210 x 82 kg/m UB (Grade S355),

Design strength p_y = 355 N/mm^2 as flange thickness T = 13.2 mm <16 mm

From section table
D = 528.3 mm, t = 9.6 mm, T = 13.2 mm, r = 12.7 mm,
d = 476.5mm, b/T = 7.91, d/t = 49.6, I_x = 47540 cm^4,
Z_x = 1800 cm^3, S_x = 2059 cm^3

Check section classification:

$$\varepsilon = \sqrt{\frac{275}{p_y}} = \sqrt{\frac{275}{355}} = 0.88$$

$$\frac{b}{T} = 7.91 \leq 9\varepsilon = 9*0.88 = 7.92, \qquad \frac{d}{t} = 49.6 \leq 80\varepsilon = 80*0.88 = 70.4$$

$$and \;\; \frac{d}{t} \leq 70\varepsilon = 70*0.88 = 61.6$$

Checking for shear web buckling is **NOT** required and the beam section is a plastic section.

Shear

Factored shear force V (at supports) = 292.3 kN

Shear area $A_v = tD$ = 9.6*528.3 = 5071 mm^2

Shear capacity $V_c = \frac{p_y \cdot A_v}{\sqrt{3}} = \frac{355*5071}{\sqrt{3}} = 1039.3kN$

$> V = 292.3$ kN

$\Rightarrow$ shear capacity O.K.

Moment:

Max. moment = 620 kNm and the co-existent shear = 62 kN

Since $V = 62$ kN $<$ $0.6\ V_c = 0.6*1039.3 = 623.6$ kN

$\Rightarrow$ No reduction in moment capacity

Moment capacity $M_c = p_y * S_x = 355*2059*10^{-3} = 730.9$ kNm

$\leq 1.2 p_y * Z_x = 1.2*355*1800*10^{-3} = 766.8$ kNm

Therefore moment capacity $M_c = 730.9$ kNm

$>$ design moment = 620 kNm

$\Rightarrow$ O.K.

Web Bearing (width of stiff bearing = 160 mm)

$k = T + r = 13.2 + 12.7 = 25.9$ mm

$n = 2 + 0.6b_e/k = 2 + 0.6*0/25.9 = 2$

Bearing Resistance $P_{bw} = (b_1 + nk)*t*p_{yw} = (160 + 2*25.9)*9.6*355$

$= 721.8$ kN > 292.3 kN $\Rightarrow$ O.K.

Web Buckling (width of stiff bearing = 160 mm)

Assume $a_e = 80$ mm,

The buckling resistance $P_x = \frac{a_e + 0.7d}{1.4d} \cdot \frac{25\varepsilon t}{\sqrt{(b_1 + nk)d}} P_{bw}$

$$= \frac{80 + 0.7*476.5}{1.4*476.5} \cdot \frac{25*0.88*9.6}{\sqrt{(160 + 2*25.9)*476.5}} * 721.8$$

$= 297.5$ kN > 292.3 kN $\Rightarrow$ O.K.

<u>Deflection δ Serviceability loads (unfactored imposed loads)</u>

P = 60 kN concentrated load at the mid-span

UDL = 35 kN/m along the whole span

$$\delta = \frac{PL^3}{48EI} + \frac{5}{384}\cdot\frac{wL^4}{EI} = \frac{(60*10^3)*7000^3}{48*205000*47540*10^4} + \frac{5}{384}\cdot\frac{35*7000^4}{205000*47540*10^4}$$

$$= 4.40 + 11.23 = 15.63\,\text{mm}$$

Assuming brittle finishes,

Allowable δ =L/360 = 7000/360 = 19.4 mm > 15.63 mm, ⇒ O.K.

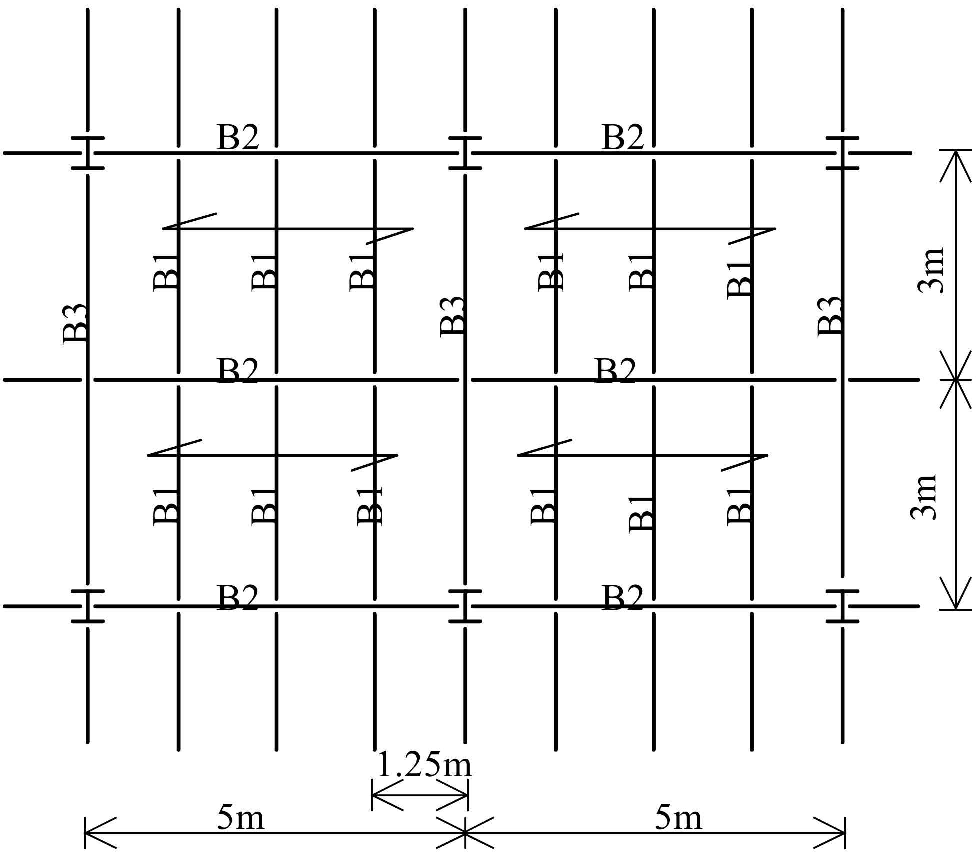
**** Alternatively one can use CONSTRADO Steelwork Design Guide Volume 1 for the design. It is time saving by using CONSTRADO design guide to carry out preliminary design. However its use is NOT covered in this course.***

Example 2 - Floor beams with compression flange laterally restrained

The steel beams for part of the floor of a library with book storage are shown in the figure below. The floor is a reinforced concrete slab on steel deck supported on universal beams. The top flanges of all beams are fully restrained laterally by the concrete slab. The design loading has been estimated as:

Dead load	=	6 kN/m^2 including self-weight of steel beams and concrete slab.
Imposed load	=	4 kN/m^2

Design beam ***B3*** by using Grade S355 steel.

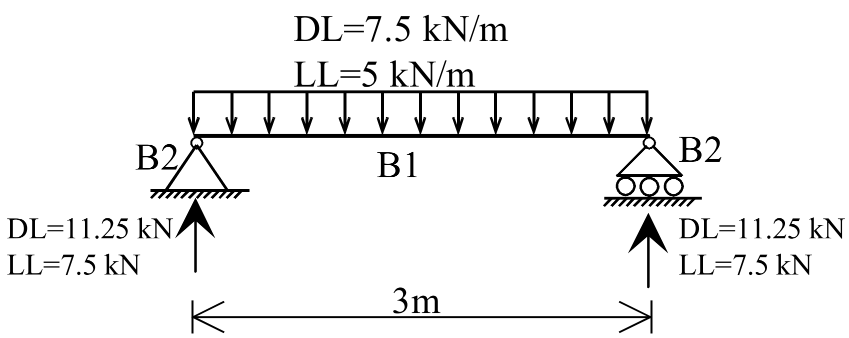


Floor Framing Plan

Solution

The UDL floor loads span one-way perpendicular to beam ***B1***. Beam ***B1*** (and ***B3***) directly supports the floor UDL and ***B2*** supports ***B1***. Beam ***B3*** supports ***B2***. Before designing beam ***B3***, we have to determine the loading on ***B3*** and hence to determine the design bending moment and shear force.

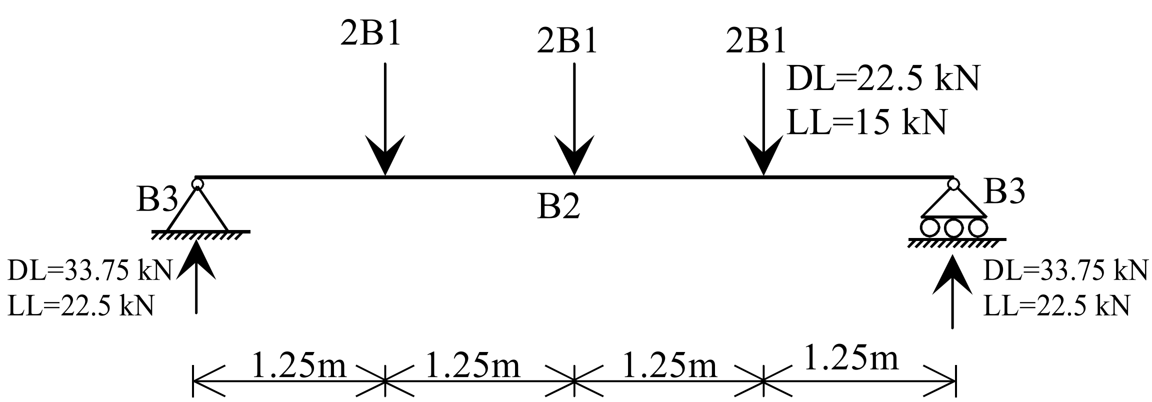
<u>Consider Beam B1 (unfactored loads), span = 3m</u>



D.L. = 6*1.25 = 7.5 kN/m
L.L. = 4*1.25 = 5 kN/m

Unfactored support reactions of B1 = 7.5*3/2 = 11.25 kN (dead)
= 5*3/2 = 7.5 kN (imposed)

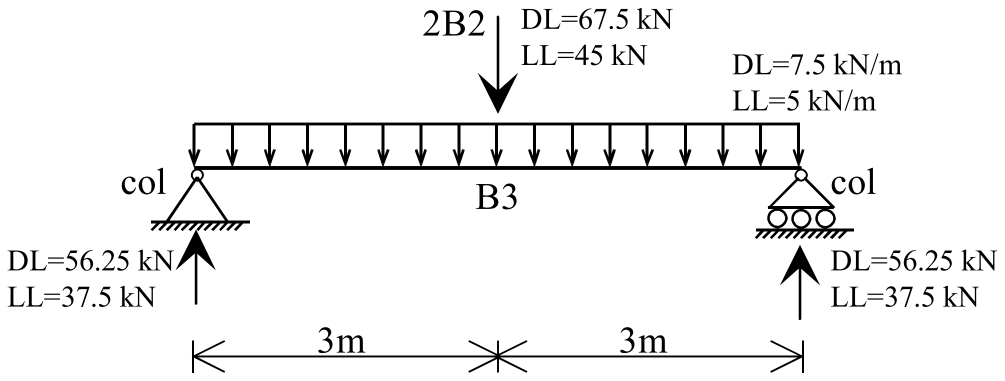
<u>Consider Beam B2 (unfactored loads), span = 5m</u>



D.L. (each point load) = 2*reaction of B1 = 11.25*2 = 22.5 kN
L.L. (each point load) = 2*reaction of B1 = 7.5*2 = 15 kN

Unfactored support reactions of B2 = 22.5*3/2 = 33.75 kN (dead)
= 15*3/2 = 22.5 kN (imposed)

Consider Beam B3 (unfactored loads), span = 6m



D.L. (point load) = 2*reaction of B2 = 33.75*2 = 67.5 kN
(UDL – slab) = 6*1.25 = 7.5 kN/m

L.L. (point load) = 2*reaction of B2 = 22.5*2 = 45 kN
(UDL – slab) = 4*1.25 = 5 kN/m

Unfactored support reactions of B3 = 7.5*6/2 + 67.5/2 = 56.25 kN (dead)
= 5*6/2 + 45/2 = 37.5 kN (imposed)

Factored point load P = 1.4*67.5 + 1.6*45 = 166.5 kN
Factored UDL = 1.4*7.5 + 1.6*5 = 18.5 kN/m
Factored reaction = 1.4*56.25 + 1.6*37.5 = 138.8 kN

Design Moment $M = \frac{Pl}{4} + \frac{wl^2}{8} = \frac{166.5*6}{4} + \frac{18.5*6^2}{8} = 333\,\text{kNm}$

Design shear force = 138.8 kN

Assume the flange thickness $\boldsymbol{T}$ is less than 16 mm, $\boldsymbol{p_y}$ = 355 N/mm^2

The plastic section modulus required,

$$S_x = \frac{M_x}{p_y} = \frac{333x10^6}{355} = 938000\,\text{mm}^3 = 938\,\text{cm}^3$$

Try 457 x 152 x 52 kg/m UB (Grade S355), Design strength $\boldsymbol{p_y}$ = 355 N/mm^2 as flange thickness $\boldsymbol{T}$ = 10.9 mm < 16 mm

From section table
D = 449.8 mm, t = 7.6 mm, T = 10.9 mm, r = 10.2 mm,
d = 407.6mm, b/T = 6.99, d/t = 53.6, I_x = 21370 cm^4,
Z_x = 950 cm^3, S_x = 1096 cm^3

Check section classification:

$$\varepsilon = \sqrt{\frac{275}{p_y}} = \sqrt{\frac{275}{355}} = 0.88$$

$$\frac{b}{T} = 6.99 \le 9\varepsilon = 9*0.88 = 7.92, \qquad \frac{d}{t} = 53.6 \le 80\varepsilon = 80*0.88 = 70.4$$

$$and \ \ \frac{d}{t} \le 70\varepsilon = 70*0.88 = 61.6$$

Checking for shear web buckling is **NOT** required and the beam section is a plastic section.

Shear

Factored shear force $\boldsymbol{V}$ (at supports) = 138.8 kN

Shear area $\boldsymbol{A_v = tD}$ = 7.6*449.8 = 3418 mm^2

Shear capacity $$V_c = \frac{p_y \cdot A_v}{\sqrt{3}} = \frac{355*3418}{\sqrt{3}} = 700.6kN$$

$> \boldsymbol{V}$ =138.8 kN

$\Rightarrow$ shear capacity O.K.

Moment:

Max. moment = 333 kNm and the co-existent shear = 83.25 kN
Since $\boldsymbol{V}$ = 83.25 kN < $\boldsymbol{0.6\ V_c}$ = 0.6*700.6 = 420.4 kN

$\Rightarrow$ No reduction in moment capacity

Moment capacity $$M_c = p_y * S_x = 355*1096*10^{-3} = 389 \text{ kNm}$$

$$\le 1.2 p_y * Z_x = 1.2*355*950*10^{-3} = 404.7 \text{ kNm}$$

Therefore moment capacity $\boldsymbol{M_c}$ = 389 kNm > design moment = 333 kNm

$\Rightarrow$ O.K.

<u>Deflection δ Serviceability loads (unfactored imposed loads)</u>

P = 45 kN concentrated load at the mid-span
UDL = 5 kN/m along the whole span

$$\delta = \frac{PL^3}{48EI} + \frac{5}{384} \cdot \frac{wL^4}{EI} = \frac{(45*10^3)*6000^3}{48*205000*21370*10^4} + \frac{5}{384} \cdot \frac{5*6000^4}{205000*21370*10^4}$$

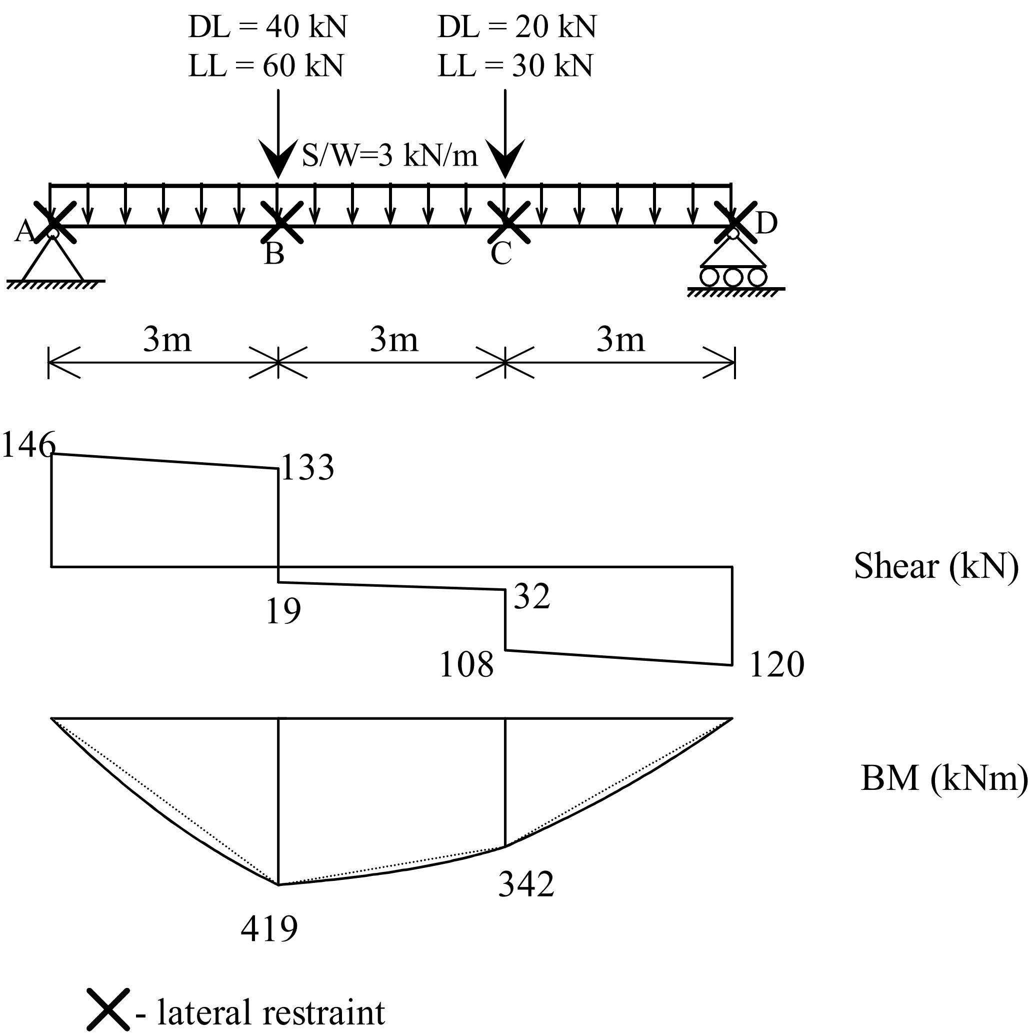
$$= 4.62 + 1.93 = 6.55\,\text{mm}$$

Assuming brittle finishes,

Allowable δ =L/360 = 6000/360 = 16.7 mm > 6.55 mm, ⇒ O.K.

Example 3
(Beam with lateral restraints at the ends and at the points of load application)

Design the steel beam for shear and bending (Take the effective length of beam segments as the length of segments).



Solution

Load Factors γ_f Dead Load = 1.4
Imposed Load = 1.6

Factored Loads P.L. $W_1 = (40 \times 1.4) + (60 \times 1.6) = 152$ kN
$W_2 = (20 \times 1.4) + (30 \times 1.6) = 76$ kN
U.D.L due to self wt. $3 \times 1.4 = 4.2$ kN/m

The design shear forces and bending moments are shown in above:
Assume 457 x 191 x 74 UB (Grade S355),
D = 457 mm, t = 9.0 mm, T = 14.5 mm, b/T = 6.57, d/t = 45.3
$r_y = 4.20$ cm, $Z_x = 1458$ cm^3, $S_x = 1653$ cm^3, u = 0.877, x=33.9

As T = 14.5mm < 16 mm, $p_y = 355$ N/mm^2
And $\varepsilon = \sqrt{\dfrac{275}{p_y}} = \sqrt{\dfrac{275}{355}} = 0.88$

Shear buckling
Since d/t = 45.3 < 70ε = 70*0.88 = 61.6,
Checking for shear web buckling is **NOT** required.

Check shear
Shear area $\boldsymbol{A_v = t\,D} = 9.0 \times 457 = 4113$ mm^2

Shear capacity $V_c = \dfrac{p_y \cdot A_v}{\sqrt{3}} = \dfrac{355 * 4113}{\sqrt{3}} = 843kN$,
> Design shear force ***V*** at ***A***= 146 kN ⇒ O.K.

Check section for combined moment and shear
Shear force at ***B*** = 133 kN
Since $\boldsymbol{V} = 133 < 0.6\,\boldsymbol{V_c} = 0.6 \times 843 = 505.8$ kN, no reduction in moment capacity.

b/T = 6.57 < 9ε = 9*0.88 = 7.92, d/t = 45.3 < 80ε = 80*0.88 = 70.4
⇒ Plastic section

$\boldsymbol{M_c} = \boldsymbol{p_y\, S_x \le 1.2\, p_y\, Z_x}$
$= 355 \times 1653 \times 10^{-3}$
$= 586 \text{ kNm} \le 1.2 \times 355 \times 1458 \times 10^{-3} = 621 \text{ kNm}$

Design moment at $\boldsymbol{B}$ = 419 kNm < $\boldsymbol{M_c}$ = 586 kNm ⇒ Section O.K.
Similarly, moment and shear at $\boldsymbol{C}$ is checked O.K.

Since the beam is subjected to possible lateral torsional buckling, the buckling resistance moment $\boldsymbol{M_b}$ should be considered. From the bending moment diagram above, $\boldsymbol{BC}$ is the critical unrestrained length. Consider that self-weight is insignificant and assume that the bending moment diagram for segment of $\boldsymbol{BC}$ as a straight line.

$$\beta = \frac{M_C}{M_B} = \frac{342}{419} = 0.82 \qquad \text{and} \qquad \boldsymbol{m_{LT}} = 0.93 \qquad \text{[Table 8.4a]}$$

Maximum moment on length $\boldsymbol{BC}$ = Moment at $\boldsymbol{B}$ = 419 kNm.
Therefore design moment $\boldsymbol{m_{LT}\,M_x}$ = 0.93 x 419 = 389.7 kNm

Buckling resistance moment $\mathbf{M_b}$

$$\boldsymbol{M_b = p_b\, S_x}$$

To find $\boldsymbol{p_b}$ one must first determine the equivalent slenderness $\boldsymbol{\lambda_{LT}}$.

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w}$$

Conservatively, $\boldsymbol{n}$ = 1.0, $\boldsymbol{u}$ = 0.9 and $\boldsymbol{v}$ =1.0 may be used. However, accurate values may be obtained as shown below :

$\boldsymbol{\lambda = L_E / r_y}$ = 3000 x 10^{-1}/4.20 = 71.4

From section book, $\boldsymbol{x}$ = 33.9 ⇒ $\boldsymbol{\lambda / x}$ = 71.4 / 33.9 = 2.11

$$v = \frac{1}{\left(1+0.05(\lambda/x)^2\right)^{0.25}} = \frac{1}{\left(1+0.05(2.11)^2\right)^{0.25}} = 0.951$$

$\boldsymbol{u}$ = 0.877 [Section Table]

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.877 * 0.951 * 71.4 * \sqrt{1} = 59.5$$

For $\boldsymbol{p_y}$ = 355 N/mm^2, Table 8.3a gives $\boldsymbol{p_b}$ =259 N/mm^2

Hence $\boldsymbol{M_b = p_b\, S_x}$ = 259 x 1653 x 10^{-3} = 428 kNm.

Since design moment $\boldsymbol{m_{LT}\,M_x}$ = 389.7 < $\boldsymbol{M_b}$ = 428 kNm
⇒ section O.K. for lateral torsional buckling

Check for bearing and buckling of the web

Students should try to check these using the same approach as in the previous example.

Deflection under serviceability loads

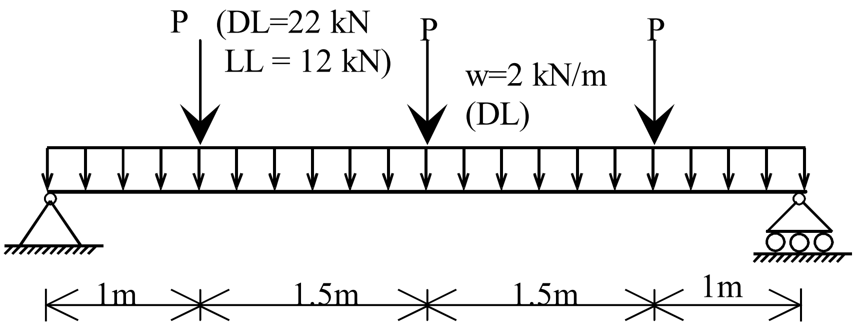
For normal structural purposes only the deflection due to the ***unfactored imposed loads*** need be considered and, deflection is not usually a critical factor unless the span/depth ratio is high. Student should try to check the deflection by themselves.

Therefore section 457 x 191 x 74 UB Grade S355 steel is adequate

Example 4 - Beam with Unrestrained Compression Flange

Design the simply supported beam for the loading shown in the figure. The loads ***P*** are normal loads. The beam ends are restrained against torsion with the compression flange free to rotate in plan. The compression flange is unrestrained between supports. Use Grade S355 steel.

Characteristic loads:
Point loads: ***P*** = 22 kN (DL)
= 12 kN (LL)
UDL ***w*** = 2 kN/m (DL)



Solution

Factored UDL = 1.4*2 = 2.8 kN/m
Factored PL(each) = 1.4*22 + 1.6*12 = 50 kN

Support Reaction = 2.8*5/2 + 3*50/2 = 82 kN

Max. moment occurs at mid-span,
$\boldsymbol{M} = 82*2.5 - 2.8*2.5^2/2 - 50*1.5 = 121.3$ kNm

Try <u>452 x 152 x 52 UB</u> (Grade S355), from section table
D = 449.8 mm, t = 7.6 mm, T = 10.9 mm, b/T = 6.99, d/t = 53.6
r_y = 3.11 cm, Z_x = 950 cm^3, S_x = 1096 cm^3, u = 0.859, x=43.9

As ***T*** = 10.9mm < 16 mm, p_y = 355 N/mm^2

And $\varepsilon = \sqrt{\dfrac{275}{p_y}} = \sqrt{\dfrac{275}{355}} = 0.88$

Shear buckling
Since ***d/t*** = 53.6 < 70ε = 70*0.88 = 61.6,
Checking for shear buckling is **NOT** required.

Check shear
Shear area $\boldsymbol{A_v = t\,D}$ = 7.6 x 449.8 = 3418 mm^2

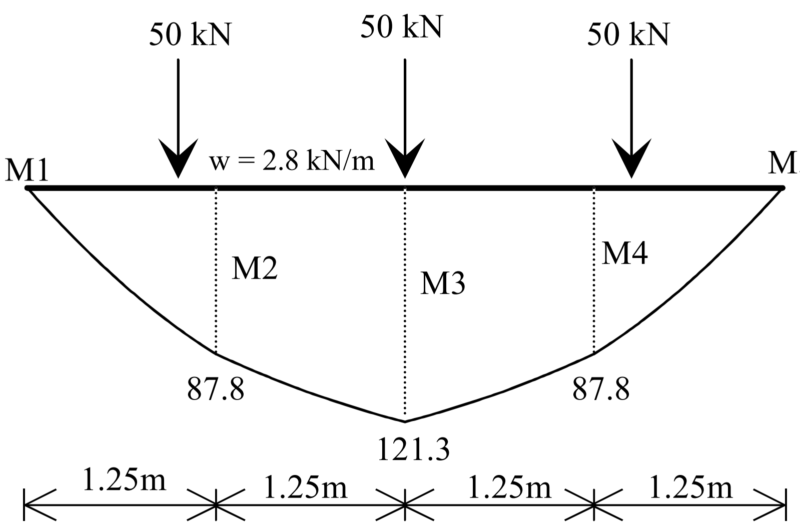
Shear capacity $V_c = \dfrac{p_y \cdot A_v}{\sqrt{3}} = \dfrac{355*3418}{\sqrt{3}} = 700.6kN$

Design shear force V at support = 82 kN $\Rightarrow$ O.K.

Lateral Torsional Buckling Check

Since the beam is subject to possible lateral torsional buckling, the buckling resistance moment $\boldsymbol{M_b}$ should be considered.

From the bending moment diagram,



From Table 8.4b,

$$m_{LT} = 0.2 + \frac{0.15M_2 + 0.5M_3 + 0.15M_4}{M_{\max}} = 0.2 + \frac{0.15*87.8 + 0.5*121.3 + 0.15*87.8}{121.3}$$

$$= 0.917 > 0.44$$

The equivalent uniform moment $\boldsymbol{m_{LT}\,M_x}$ should be calculated and must not exceed $\boldsymbol{M_b}$.

$\boldsymbol{m_{LT}\,M_x}$ = 0.917*121.3 = 111.2 kNm

<u>Buckling resistance moment M_b</u>

$\boldsymbol{M_b = p_b\, S_x}$

To find $\boldsymbol{p_b}$ one must first determine the equivalent slenderness $\boldsymbol{\lambda_{LT}}$.

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w}$$

Accurate values may be obtained as shown below :

$\boldsymbol{\lambda = L_E / r_y}$ = 5000 /31.1 = 160.8

From section book, $\boldsymbol{x}$ = 43.9 $\Rightarrow$ $\boldsymbol{\lambda / x}$ = 160.8 / 43.9 = 3.66

$$v = \frac{1}{\left(1 + 0.05(\lambda / x)^2\right)^{0.25}} = \frac{1}{\left(1 + 0.05(3.66)^2\right)^{0.25}} = 0.880$$

$\boldsymbol{u}$ = 0.859 [Section Table]

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w} = 0.859 * 0.880 * 160.8 * \sqrt{1} = 121.6$$

For $\boldsymbol{p_y}$ = 355 N/mm^2 Table 8.3a gives $\boldsymbol{p_b}$ =101.8 N/mm^2

Hence $\boldsymbol{M_b = p_b\, S_x}$ = 101.8 x 1096 x 10^{-3} = 111.6 kNm.

Since design moment $\boldsymbol{m_{LT}\, M_x}$ = 111.2 kNm < M_b = 111.6 kNm

$\Rightarrow$ section O.K. for lateral torsional buckling

🕮 Revision

Read reference 2 on P.24 -54.

🕮 Main Reference

1. *Code of practice for Structural Use of Steel 2011, Buildings Department, the Government of HKSAR*
2. *Structural Steelwork, Design to Limit State Theory, 3rd edition (2004), Dennis Lam, Thien-Cheong Ang, Sing-Ping Chiew, Elsevier.*
3. *Limit States Design of Structural Steelwork, 3rd edition (2001), D.A. Nethercot, Spon Press.*
4. *The Behaviour and Design of Steel Structures to BS5950, 3rd edition (2001), N.S. Trahair, M.A. Bardford, D.A. Nethercot, Spon Press.*
5. *Steel Designers' Manual, 6th edition (2003), Oxford: Blackwell Science, Steel Construction Institute.*
6. *Structural Steelwork, Design to Limit State Theory, 2nd edition, T.J. MacGinley and T.C. Ang, Butterworths.*

| TUTORIAL 3 |

Q1. The cross sections of steel beams are classified into classes 1, 2, 3 & 4 as plastic, compact, semi-compact, and slender cross sections respectively in accordance with their behavior in bending,. Briefly describe their behavior in bending

Q2. Figure Q2 shows a part plan of a braced steel structure. The concrete slab is a solid RC slab of 130 mm thick. The compression flanges of steel beams are fully restrained by the concrete slab. All the steel beams are grade S355 and simply supported. It is given that the concrete slab is subjected to the following characteristic loads:-

(a) Characteristic Dead Load:

Self-weight of slab

Floor finishes = 1 kN/m^2

Partition load = 1 kN/m^2

Services = 0.5 kN/m^2

(b) Characteristic Imposed Load = 5 kN/m^2

Check the adequacy of the steel beams with respect to bending, shear and deflection (the beams has to support brittle finishes). Also check the web bearing and web buckling resistance of beam ***B3*** if the stiff bearing widths at the supports are 150 mm and the flange is effectively restrained against both rotation relative to the web and lateral movement relative to other flange. Take a_e = 75 mm and b_e = 0.

The sizes of the steel beams are as follows:

B1: 406 x 140 x 46 kg/m UB

B2: 457 x 191 x 67 kg/m UB

B3: 457 x 191 x 67 kg/m UB

Q3. Refer to question Q2. If full lateral restraint is **NOT** provided by the concrete slab, check the adequacy of the beam sections of ***B2*** and ***B3*** in regard to Lateral Torsional Buckling. In checking ***B3***, lateral restraints are assumed to be provided by beam ***B2*** and at the supports. The effective lengths of members are assumed to be the actual segment lengths and the loads are normal loads.

Q4. A simply supported steel beam of 533 x 210 x 82 kg/m UB (grade S355) is subjected to a design end moment M of 350 kNm and a design point load P of 180 kN as shown in Figure Q4. Lateral restraints are provided at the supports and at the point load position. Check the adequacy of the beam section against lateral torsional buckling assuming the actual segment lengths are the effective lengths and the loads are normal loads.

| TUTORIAL 3 |

Q5. It is required to design a steel beam with an overhanging segment. The dimension and loading (including the self-weight of the beam) are shown in Figure Q5. The beam has torsional restraints at the supports but no intermediate lateral support. Select a suitable UB using Grade S355 steel. You may assume the effective lengths of the beam are equal to the actual length of segements.

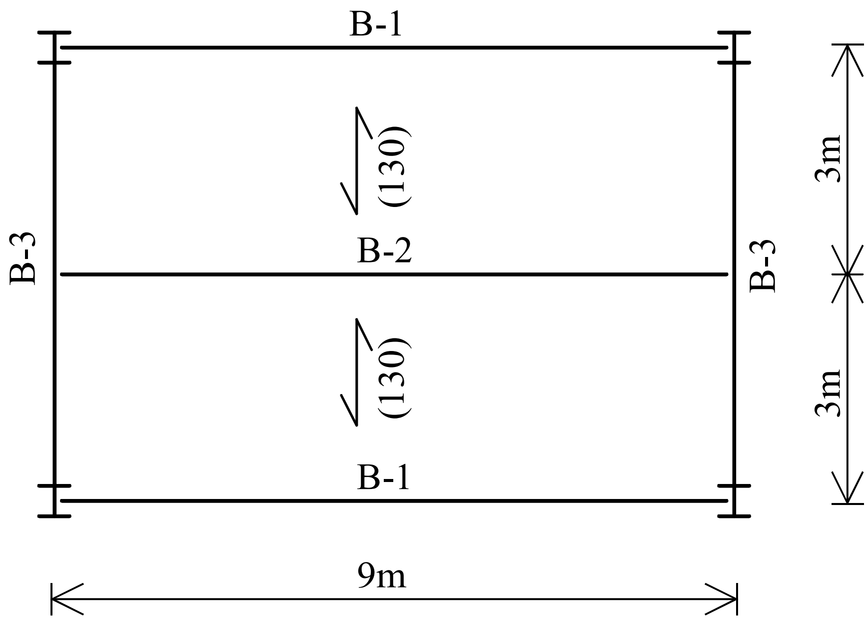


Figure Q2

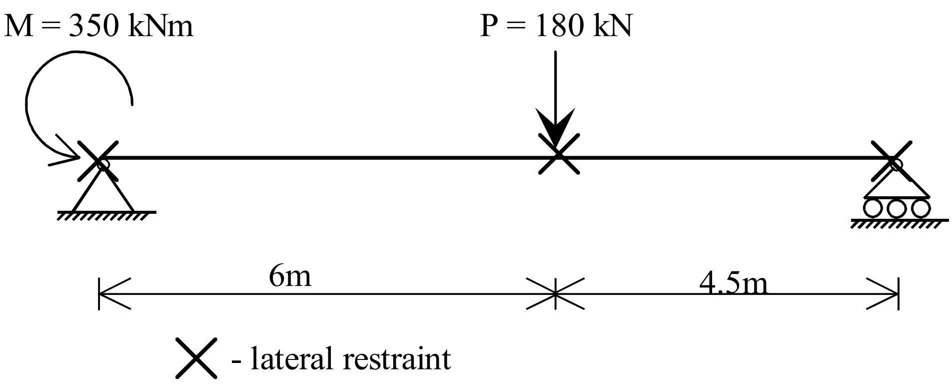
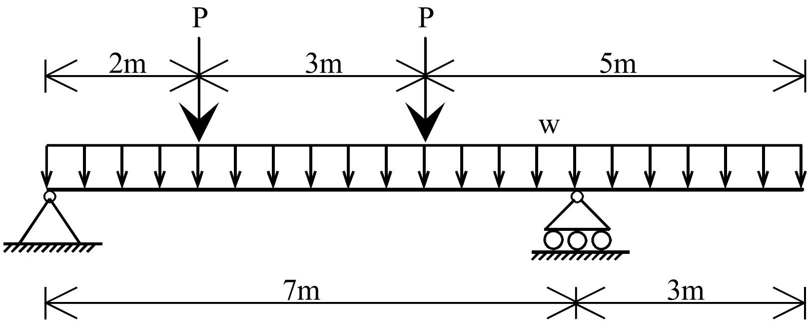


Figure Q4

| TUTORIAL 3 |



For P, DL = 60 kN and LL = 35 kN
For w, DL = 10 kN/m and LL = 5 kN/m

Figure Q5